Parabolic Dynamical Systems (9 ECTS)
Giovanni Forni
2e semestre

Présentation

The lectures will be focused on the dynamics of weakly chaotic (zero entropy) systems which are relevant as simplified models of physical systems or in applications to geometry and number the- ory. We will examine several examples, including translation flows and billiards in polygons and related systems, horocycle flows, nilflows and their time-changes, from a unified perspective based on renormalization and rescaling methods. We will address question of effective ergodicity, relevant in connection with number theory, limit distributions of ergodic averages, mixing and spectral properties.

The **first part** of the lecture series will be devoted to cohomological equations and invariant dis- tributions. In this part we will develop the essential analytical tools. For homogeneous flows, the analysis is based on the theory of unitary representations for the relevant Lie group. We plan to exam- ine in detail the case of niflows.
For translation flows we will outline our harmonic analysis solution (based on classical results on boundary values of holomorphic functions) which has applications to other related problems (twisted cohomological equations, non-rational polygonal billiards).
We plan to outline recent work of Giulietti and Liverani [GL] and especially Faure, Gouë zel and Lanneau [FGL] for invariant directions of pseudo-Anosov diffeomorphisms based on methods from hyperbolic dynamics (anisotropic Banach spaces, spectral gaps for transfer operators). We also plan include a discussion of the relation between obstructions to existence of solutions of cohomological equation (invariant distributions) for unstable flows and Ruelle resonances [Fo4].

The **second part** of the lecture series will be devoted to effective equidistribution results (and rela- tions with number theory). We will present a renormalization/rescaling approach originally inspired to the work of Kontsevich and Zorich on Interval Exchange Tranformations and translation flows. This approach directly generalizes to several renormalizable flows (billiard flows in rational poly- gons [AtF], horocycle flows [FF], [Fo2], Heisenberg nilflows [Fo3]). For higher step nilflows [Fo3], related to Weyl sums for higher degree polynomials, and “twisted” horocycle flows [FFT] (related to Fourier coefficients of cusp forms), which are not renormalizable, effective equidistribution results can still be derived by a scaling approach (renormalization without moduli space).

In the **third part** of the lecture series, we plan to discuss ergodic-theoretical questions such as limit distributions for translation flows, horocycle flows and Heisenberg nilflows (following work of Bufetov [Bu] on translation flows, joint work with Bufetov [BF], [Fo2] in the horocycle case), mixing, decay of correlations and spectral properties for translation flows, and for time-changes of homogeneous flows (following joint work with Avila and Ulcigrai [AFU] and with Kanigowski [FK] on nilflows, with Ulcigrai [FU] on horocycle flows, and with Fayad and Kanigoswki [FFK] on surface flows).

Throughout the lectures we plan to emphasize open questions and possible research directions, for examples related to analytical methods coming from hyperbolic dynamics in the study of cohomolog- ical equations and effective equidistributions, to the generalization of the theory of translation flows to higher dimensions, to the spectral theory of interval exchange transformations and translation flows, to the generalization of the results to other similar systems (unipotent flows, billiards in polygons).

Bibliographie

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