AN INTRODUCTION TO ALGEBRAIC K-THEORY

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Classically, algebraic K-theory of rings is the study of the family of K-theory functors

$$K_n : \text{ Rings } \to \text{ Abelian groups}, \quad (n = 0, 1, 2)$$

that can be defined algebraically, and that are closely related to arithmetic invariants of commutative rings, such as the Picard, unit and Brauer groups. In the early 70's, several definitions of higher K-groups were proposed; the most striking one was Quillen's definition, which uses categories and homotopy theory: he defined the group K_nR as the n-th homotopy group of a certain *algebraic K-theory space* or *spectrum* KR :

$$K_nR = \pi_n(KR) \quad (n = 0, 1, 2, \dots).$$

The space KR is defined from the category of projective R-modules of finite type by a group-completion process for the direct sum of modules.

The purpose of this lecture course is to introduce *higher algebraic K-theory* in the sense of Quillen, including the necessary tools from category theory and homotopy theory, and discuss some examples and applications.

Prerequisites: basic topology, elementary algebraic topology, homological algebra (for example the lecture of Livernet).

References

[ML98] Saunders Mac Lane, *Categories for the working mathematician*, Second, Graduate Texts in Mathematics, vol. 5, Springer-Verlag, New York, 1998.

[Qui73] Daniel Quillen, *Higher algebraic K-theory. I*, Lecture Notes in Math., Vol. 341, 85–147, 1973.

[Ros94] Jonathan Rosenberg, *Algebraic K-theory and its applications*, Graduate Texts in Mathematics, vol. 147, Springer-Verlag, New York, 1994.

[Sri96] V. Srinivas, *Algebraic K-theory*, Second, Progress in Mathematics, vol. 90, Birkhäuser Boston, Inc., Boston, MA, 1996.

[tD08] Tammo tom Dieck, *Algebraic topology*, EMS Textbooks in Mathematics, European Mathematical Society (EMS), Zürich, 2008.

[Wal85] Friedhelm Waldhausen, *Algebraic K-theory of spaces*, Lecture Notes in Math. 1126, 318–419, Springer, Berlin, 1985.

[Wei13] Charles A. Weibel, *The K-book*, Graduate Studies in Mathematics, vol. 145, American Mathematical Society, Providence, RI, 2013. An introduction to algebraic K-theory.