Homotopy Theory (9 ECTS)

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1er semestre

Program

The goal of this lecture will be to present various "concrete" homotopy theories. We will start with the classical homotopy theory of topological spaces (higher homotopy groups, cellular complexes, Whitehead and Hurewicz theorems, Eilenberg—MacLane spaces, fibrations, and Postnikov towers). Then we will move to the homotopy theory of simplicial sets (definitions, simplex category, adjunction and cosimplicial objects, examples, fibrations, Kan complexes, and simplicial homotopy). Finally, we will study the rational homotopy theory via the homotopy theory of differential graded Lie or commutative (co)algebras (Sullivan approach : minimal model, Quillen approach : Whitehead Lie bracket, bar and cobar constructions, complete Lie algebra-Hopf algebras-groups).

This presentation opens the door to the axiomatic treatment of homotopy theory done by Quillen and treated in details in the next course « Homotopical Algebra » (Grégory Ginot, Paris 13).

Prerequistes

Basic notions of category theory, chain complexes, exact sequence, singular (co)homology (all being covered by the previous course « Homology Theory »).

References

[FHT01] Y. Félix, S. Halperin, and J.-C. Thomas. ***Rational homotopy theory***, Graduate Texts in Mathematics, 205, Springer, 2001.

[GJ99] P. G. Goerss and J. F. Jardine, ***Simplicial homotopy theory***, reprint of the 1999 edition, Birkhauser, 2009.

[GM13] P. Grifiths, J. Mornan, ***Rational Homotopy Theory and Differential forms***, second edition, Birkhauser, 2013.

[Hat02] A. Hatcher, ***Algebraic Topology***, Cambridge University Press, Cambridge, 2002.

[Whi78] G. Whitehead, ***Elements of homotopy theory***, Graduate Texts in Mathematics,61. Springer-Verlag, New York-Berlin, 1978.